COMBINATION OF DOPPLER NETWORKS WITH TERRESTRIAL NETWORKS

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ABSTRACT:

For the combination of Doppler networks with terrestrial one, two mathematical models are completely derived and introduced. The first model is for three dimensional combination and the second for two dimensional combination, results from the two mathematical models show that, as long as we do not have accurate informations in Egypt about the orthometric and normal heights of the terrestrial network, then the two dimensional combination will be preferable because it is much more easy to be handeld.

INTRODUCTION:

The combination of a Doppler network with a terrestrial one serves generally to improve the quality of the inner strength of the terrestrial network and to determine the transformation parameters for the positioning of the reference ellipsoid in a geocentric system. It is clear that we are dealing with different coordinate systems. Accordingly, a direct comparison between the coordinates of the Doppler and geodetic stations cannot be done until the relation between these systems are known. The transformation from a given geodetic reference to the Doppler system , are controlled by certain number of parameters, [3], [4], [5], these parameters are mainly translation, rotation & scale.

2. THREE DIMENSIONAL COMBINATION:

2.1 Preliminary transformation of Doppler network:

The Doppler coordinates used for the preliminary transformation from a geocentric system to a local system will include all the common stations. Let $\underline{r}_{iD}$ and r_{it} denote the Doppler and terrestrial coordinates of the i-th station , respectively.

$$\underline{r}_{iD} = \left[\underline{x}_{iD} , \underline{y}_{iD} , \underline{z}_{iD} \right]^T \quad (i = 1,2,3,...n) \qquad (2.1)$$

and

$$\underline{r}_{D} = \left[..... , \underline{r}_{iD}^T , ... \right]^T \qquad (2.2);$$

$$r_{it} = \left[x_{it} , y_{it} , z_{it} \right]^T = \begin{bmatrix} (N_i+H_{it}) \cos B_{it} \cos L_{it} \\ (N_i+H_{it}) \cos B_{it} \sin L_{it} \\ (N_i(1-e^2)+H_{it}) \sin B_{it} \end{bmatrix} \qquad (2.3);$$

and

$$r_t = \left[........., r_{it}^T , ... \right]^T \qquad (2.4).$$

where B_{it} and L_{it} are the estimators obtained from the previous adjustment of the terrestrial network before the combination with the Doppler network. The geodetic height H_{it} of the i-th station can be obtained from the height anomaly and normal height of the i-th station. The misclosure between $\underline{r}_D$ and r_t can be denoted by

$$d = r_t - \underline{r}_D \qquad (2.5);$$

The preliminary transformation parameters are restricted to only three shift parameters $\underline{r}_o$

$$\underline{r}_o = \left[x_o \ , \ y_o \ , \ z_o \ \right]^T \qquad (2.6);$$

From expression (2.5) and (2.6) the observation equations for the preliminary transformation are obtained

$$Z\underline{r}_o + d = v \qquad (2.7);$$

where

$$Z = \left[I, I, \ldots \ \right]^T \quad (I = \text{identity matrix}) \qquad (2.8);$$

The three preliminary shift parameters can be obtained from the following equation:

$$\underline{r}_o = (Z^T Z)^{-1}(Z^T d) = -(Z^T d)/n \qquad (2.9);$$

Thus we obtain the transformed Doppler coordinates r_D.

$$\begin{aligned} r_D &= \underline{r}_D + Z\underline{r}_o \\ &= \left[\ldots\ldots\ldots\ldots, \ r^T_{iD}, \ \ldots\ldots \ \right]^T \qquad (2.10); \\ &= \left[\ldots\ldots\ldots\ldots, x_{iD}, t_{iD}, z_{iD}, \ldots\ldots\ldots \right]^T \end{aligned}$$

r_D obtained from formula (2.10) differ from the terrestrial coordinates r_t only by a small amount.

2.2 Observation equations for the combination of the Doppler network with the terrestrial one:

In order to minimize the effect of the error resulting from the Doppler system on the inner strength of terrestrial network, only a restricted number of Doppler stations-reliable accuracy, widely separation shall be used.

Doppler results originally are expressed in terms of an x,y,z - space cartesian coordinate system,however the station coordinates in the terrestrial network of Egypt are expressed in geodetic latitudes, longitudes and hiehgts (B,L,H) System. So that a transformation between both systems bec omes indispensable in a combined solution. The transformation of the Doppler coordinates x,y,z to B,L, H system is therefore more expedite than the reverse transformation of the terrestrial network.

Using the inverse problem [1], [2], [5] the corresponding geodetic coordinates B_{iD}, L_{iD}, H_{iD} of the i-th Doppler station can be obtained from x_{iD}, y_{iD}, z_{iD} .

Now we introduce a new vector dR. They correspond to the corrections for the geodetic coordinates of Doppler common stations after the preliminary transformation.

$$dR = \left[\ldots\ldots, dR_i^T , \ldots \right]^T$$
$$dR_i^T = \left[(M_i + H_i) dB_i , (N_i + H_i) \cos B_i dL_i , dH_i \right]^T \qquad (2.11).$$

The coordinates of the common stations B_{iD}, L_{iD}, H_{iD} obtained from Doppler positioning are considered as the quasi observations of those in the terrestrial network. The observation equations of the combination of the Doppler network with the terrestrial one can be established as follows.

$$dR - C\,Z\,dr_o + C\,r_D\,dk - C\,F\,DE + W = v \qquad (2.12);$$

where

$$dr_o = \left[dx_o\,,\ dy_o\,,\ dz_o \right]^T \qquad (2.13);$$

$$C = \mathrm{diag}\left[\ldots\ldots\,,\ C_i\,,\ \ldots\ldots \right] \qquad (2.14);$$

$$C_i = \begin{bmatrix} -\sin B_{iD}\cos L_{iD}, & -\sin B_{iD}\sin L_{iD}, & \cos B_{iD} \\ -\sin L_{iD}\,, & \cos L_{iD}\,, & 0 \\ \cos B_{iD}\cos L_{iD}, & \cos B_{iD}\sin L_{iD}, & \sin B_{iD} \end{bmatrix} \qquad (2.15);$$

$$F = \left[\ldots\ldots,\ F_i^T\,,\ldots\ldots \right]^T \qquad (2.16);$$

$$F_i = \begin{bmatrix} 0\,, & z_{iD}, & -y_{iD} \\ -z_{iD}, & 0\,, & x_{iD} \\ y_{iD}, & -x_{iD}, & 0 \end{bmatrix}$$

$$dE = \left[dE_x,\ dE_y,\ dE_z \right]^T \qquad (2.17);$$

$$W = \left[\ldots\ldots,\ W_i^T\,,\ \ldots\ldots \right]^T$$

$$W_i = \left[(M_i+H_i)(B_{it}^o - B_{iD}),\ (N_i+H_i)\cos B_{it}(L_{it}^o - L_{iD}),\ (H_{it}^o - H_{iD}) \right]^T \qquad (2.18)$$

where

B_{it}^o, L_{it}^o, H_{it}^o are initial values taken from a previous adjustment of the terrestrial network.

2.3 Weight matrices for the parameters in the observation equations:

(a) Weighting the quasi - observations:

The quasi - observations B_{iD}, L_{iD} and H_{iD} in formula (2.18) are the transformed coordinates r_D(see formula (2.10), which are obtained from $\underline{r}_D$ (see formula (2.2) and $\underline{r}_D$ are the adjusted coordinates of the common stations in the Doppler network. The weight coefficient matrix of $\underline{r}_D$ can be obtained from the coefficient matrix of the adjusted Doppler net.

Assuming that the weight coefficient matrix for r_D is the same as for $\underline{r}_D$ and is equal to N_D^{-1} .

According to the propagation law of covariance function the weight coefficient matrix of quasi observation in formula (2.19) is

$$Q = C\, N_D^{-1} C^T$$

Inverting the above weight coefficient matrix and taking into account $C^{-1} = C^T$, the a priori weight matrix for the quasi observation in formula (2.18) is

$$p = C^T\, N_D\, C \qquad (2.19).$$

(b) Weighting the orientation parameters:

For the orientation of the final combined system two information sources are available: one from the laplace azimuths given by the terrestrial network, and the other one from the absolute coordinate given by the Doppler network. However,both information sources must be in concurrence,in other words,the orientation of the final combined system is determined by the cooperation of both networks and neither by the terrestrial network alone nor by the Doppler network alone.

Therefore a certain weight matrix P_E can be allotted to the orientation of the Doppler network,obtained from the terrestrial and the Doppler networks respectively.

(c) Weighting the scale factor:

If no correlation between scale and orientation is assumed in both networks, the same procedure mentioned above for the orientation parameters can be applied for the scale factor. The weight of the scale factor for the Doppler network, P_K, is allotted also in accordance with the ratio between the accuracies of scale factors in the terrestrial and the Doppler networks respecti.ely.

2.4 Normal equations for the combination of terrestrial networks with Doppler system:

According to the observation equation (2.12)and the weight matrices P, P_E, P_K . The normal equations are formed as follows:

$$\begin{bmatrix} P, & -PCZ, & PCr_D, & -PCF \\ -Z^TC^TP, & Z^TC^TPCZ, & -Z^TC^TPCr_D, & Z^TC^TPCF \\ r_D^TC^TP, & -r_D^TC^TPCZ, & r_D^TC^TPCr_D+P_k, & -r_D^TC^TPCF \\ -F^TC^TP, & F^TC^TPCZ, & -F^TC^TPCr_D, & F^TC^TPCF+P_E \end{bmatrix} \begin{bmatrix} dR \\ dr_o \\ dK \\ dE \end{bmatrix} + \begin{bmatrix} PW \\ -Z^TC^TPW \\ r_D^TC^TPW \\ -F^TC^TPW \end{bmatrix} = 0 \quad (2.20);$$

introducing abbreviations N_{ij}, we get

$$\begin{bmatrix} N_{DD}, & N_{Dr}, & N_{DK}, & N_{DE} \\ N_{Dr}, & N_{rr}, & N_{rK}, & N_{rE} \\ N_{UK}, & N_{rK}, & N_{KK}, & N_{KE} \\ N_{UK}, & N_{rK}, & N_{KK}, & N_{KE} \end{bmatrix} \begin{bmatrix} dR \\ dr_o \\ dK \\ dE \end{bmatrix} + \begin{bmatrix} W_D \\ W_r \\ W_k \\ W_E \end{bmatrix} = 0 \quad (2.21);$$

The adjustment of the terrestrial network is mostly carried out by the block solution. The common Doppler stations follow the connecting points of the block and are located in the rear part of the normal equations of the terrestrial network. By reducing the normal equations (include the connecting points of the block), the reduced normal equations only related to the Doppler common stations are obtained as follows.

$$N^*_{tt} \quad dR^* + W^*_{tt} = 0 \quad (2.22);$$

where

$$N^*_{tt} = \begin{bmatrix} \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots, N_{B_iB_i}, N_{B_iL_i}, \cdots\cdots \\ \cdots\cdots, N_{L_iB_i}, N_{L_iL_i}, \cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \end{bmatrix} \quad (2.23).$$

$$dR^* = \left[\;\ldots\ldots , \; (M_i+H_i)\, dB_i , \; (N_i+H_i) \cos B_i \; dL_i , \ldots\ldots \right]^T \qquad (2.24);$$

$$W_t^* = \left[\;\ldots\ldots , \; W_{B_i} , \; W_{L_i} , \; \ldots\ldots \right]^T \qquad (2.25).$$

There are no height elements in the above reduced normal equations,if a two dimensional adjustment of the terrestrial network is applied. In order to combine it with the Doppler network the coefficient matrix (2.23), the vector of the unknowns (2.24), and of the misclosure (2.25) should be filled by the pertaining terms for the height of the stations. The coefficient matrix(2.23) then is changed as follos:

$$N_{tt} = \begin{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\ \ldots\ldots , & N_{B_iB_i} , & N_{B_iL_i} , & 0 , \ldots \\ \ldots\ldots , & N_{L_iB_i} , & N_{L_iL_i} , & 0 , \ldots \\ \ldots\ldots , & 0 , & 0 , & P_{H_{it}} , \ldots \end{bmatrix} \qquad (2.26)$$

where $P_{H_{it}}$ is the weight for the geodetic height (H_{it}) of the i-th station and can be estimated in accordance with the accuracy itself.

$$W_t = \left[\;\ldots\ldots , \; W_{B_{it}} , \; W_{L_{it}} , \; 0 , \; \ldots\ldots \right]^T \qquad (2.27).$$

The final normal equations for the combined soulution are as follows:

$$\begin{bmatrix} (N_{DD}+N_{tt}) , & N_{Dr} , & N_{DK} , & N_{DE} \\ N_{Dr} , & N_{rr} , & N_{rK} , & N_{rE} \\ N_{DK} , & D_{rK} , & N_{KK} , & N_{KE} \\ N_{DE} , & N_{rE} , & N_{KE} , & N_{EE} \end{bmatrix} \begin{bmatrix} dR \\ dr_o \\ dK \\ dE \end{bmatrix} + \begin{bmatrix} (W_D+W_t) \\ W_r \\ W_K \\ W_E \end{bmatrix} = 0 \qquad (2.28)$$

2.5 The results of the combination:

The final coordinates of the Doppler common stations in the terrestrial network after the combination are

$$R_{tD} = R^{o} + dR$$

where R^{o} are the initial approximate values chosen from a previous adjustments of the terrestrial network.

$$R^{o} = \left[\ldots\ldots, R_{i}^{oT}, \ldots\ldots \right]^{T}$$

$$R_{i}^{o} = \left[(M_{i} + H_{i})B_{it}^{o}, (N_{i} + H_{i}) \cos B_{it}^{o} L_{it}^{o}, H_{it}^{o} \right]^{T} \qquad (2.29);$$

dR are obtained from the normal equations(2.28) and R_{tD} are expressed as follows

$$R_{tD} = \left[\ldots\ldots, R_{itD}^{T}, \ldots\ldots \right]^{T}$$

$$R_{itD} = \left[(M_{i}+H_{i})B_{itD}, (N_{i}+H_{i})\cos B_{itD} L_{itD}, H_{itd} \right]^{T} \qquad (2.30).$$

$$= R_{i}^{o} + dR_{i}$$

$$= \left[(M_{i}+H_{i})(B_{it}^{o}+dB_{i}), (N_{i}+H_{i}) \cos B_{itD}(L_{it}^{o} +dL_{i}), (H_{it}^{o} + dH_{i}) \right]$$

The final coordinates of the other stations, except the Doppler common stations in the terrestrial net are determined by back substitution of R_{tD}.

2.6 The final transformation parameters for the geocentric system:

After the accomplishment of the combined adjustment the transformation from the local datum of the terrestrial network to the geocentric one can be performed.

The seven transformation parameters can be determined from formulae (2.9) and (2.28).

$$r_o = \underline{r}_o + dr_o$$
$$K = dK \qquad (2.31);$$
$$E = dE$$

In the author's opinion the combined adjustment should be performed by such a limited number of Doppler stations, for which the accuracy is reliable and appropriate to the accuracy of the terrestrial network. But the determination of the final transformation parameters should be achieved using all Doppler stations in order to describe the geometric relationship between the two coordinate systems. The more Doppler stations are introduced, the higher the accuracy of the transformation parameters will be. Therefore it is recommended that we compare all the original Doppler coordinates $\underline{r}_D$ with the final coordinates of all Doppler stations in the terrestrial network; from the differences between the two kinds of coordinates we can calculate the final transformation parameters.

2.7 Weights in the Doppler network:

If the scale and the orientation of the Doppler system are desired to be maintained in the combined adjustment, then, P_K and P_E can be assumed as infinite quantities. In this case only $\underline{dr}_o$ in the normal equations (2.28) requires to be solved. If someone wishes to maintain all the coordinates of Doppler network in the combination, those coordinates can be substituted into the terrestrial network and taken as the fixed stations.

* The above two ways of combination are not recommended,t are equivalent to the well known practice in the old triangula- tion computation, where the 2^{nd} order network has to accept full constraint from the 1^{st} order triangulation. The disadvan- tages of such a procedure are sufficiently known.

3. TWO DIMENSIONAL COMBINATION:

Two dimensional combination can be performed in the follo steps 6 . The first step is the preliminary transformation of the Doppler network as described in paragraph (2.1). The second step is to establish the observation equations for the two - dimensional combination. In order to eliminate the height elements in the combined adjustment we at first reduce the Doppler coordinates of the common stations to the surface of the reference ellipsoid which is used in the adjustment of the terrestrial network. The reduced Doppler coordinates on the ellipsoid are:

$$r_{eD} = \left[\ldots\ldots , r^T_{ieD}, \ldots\ldots \right]^T \qquad (3.1).$$

$$r_{ieD} = \left[x_{ieD}, y_{ieD}, z_{ieD} \right]^T$$
$$= \left[N_i \cos B_{iD} \cos L_{iD}, N_i \cos B_{iD} \sin L_{iD}, N_i (1-e^2) \sin B_{iD} \right]^T \qquad (3.2).$$

Now the following observation equations can be listed:

$$dR'_e - CZdr_o + Cr_{eD} dK - CF_e dE + W_e = v \qquad (3.3);$$

where

$$dR'_e = U \; dR \qquad (3.4);$$

$$U = \text{diag} \left[\ldots\ldots , U_i, \ldots\ldots \right] = U^T$$

$$U_i = \begin{bmatrix} M_i/(M_i+H_i) , & 0 & ,0 \\ 0 & , N_i/(N_i+H_i) & ,0 \\ 0 & , \quad 0 & ,1 \end{bmatrix} \qquad (3.5);$$

$$F_e = \begin{bmatrix} \ldots\ldots , F_{ei}^T , \ldots\ldots \end{bmatrix}^T$$

$$F_{ei} = \begin{bmatrix} 0 & , z_{ieD} , & -y_{ieD} \\ -z_{ieD} & , \quad 0 , & x_{ieD} \\ y_{ieD} , & -x_{ieD} , & 0 \end{bmatrix} \qquad (3.6);$$

$$W_e = U\ W \qquad (3.7)$$

In formula (3.7) matrix W is taken from formula(2.18). For the weight of the quasi-observations in formula(3.7) we obtain the following formula similar to formula (2.19).

$$P_e = \left[U\ C\ N_D^{-1}\ C^T\ \ U^T \right]^{-1} \qquad (3.8)$$

$$= U^{-1} C^T N_D\ C\ U^{-1}$$

The third step is to establish the normal equations corresponding to the observation equations(3.3). They are similar to the normal equations (2.20) and presented by the following formula:

$$\begin{bmatrix} P_e & , -P_e CZ & , P_e C\ r_{eD} & , -P_e CF_e \\ -z^T C^T P_e & , z^T C^T P_e CZ & , -Z^T C^T P_e C r_{eD} & , Z^T C^T P_e CF_e \\ r_{eD}^T C^T P_e & , -r_{eD}^T C^T P_e CZ & , r_{eD}^T\ C^T P_e C r_{eD} + P_K & , -r_{eD}^T C^T P_e CF_e \\ -D_e^T C^T P_e & , F_e^T C^T P_e CZ & , -F_e^T C^T P_e C r_{eD} & , F_e^T C^T P_e CF_e + P_E \end{bmatrix} \begin{bmatrix} dR'_e \\ dr_o \\ dK \\ dE \end{bmatrix} + \begin{bmatrix} P_e W_e \\ -Z^T C^T P_e W_e \\ r_{eD}^T C^T P_e W_e \\ F_e^T C^T P_e W_e \end{bmatrix} = 0 \qquad (3.9)$$

We eliminate, by the Gaussion algorithm for instance, all dH_i (i = 1,2,....) in dR'_e and get the following reduced normal equations:

$$\begin{bmatrix} N_{eDD}, & N_{eDr}, & N_{eDK}, & N_{eDE} \\ N_{eDr}, & N_{err}, & N_{erK}, & N_{erE} \\ N_{Dk}, & N_{erK}, & N_{eKK}, & N_{eKE} \\ N_{eDE}, & N_{erE}, & N_{eKE}, & N_{eEE} \end{bmatrix} \begin{bmatrix} dR_e \\ dr_o \\ dK \\ dE \end{bmatrix} + \begin{bmatrix} W_{eD} \\ W_{er} \\ W_{eK} \\ W_{eE} \end{bmatrix} = 0 \qquad (3.10)$$

here, dR_e does not contain the height corrections dH_i

$$dR_e = \left[\ldots\ldots, \quad M_i\, dB_i, \; N_i \cos B_i\, dL_i \quad , \ldots\ldots \right]^T \qquad (3.11)$$

The fourth step is to combine the two networks. The final normal equations for the combined adjustment are similar to the equations (2.28).

$$\begin{bmatrix} N_{eDD}+N^*_{tt}, & N_{eDr}, & N_{eDK}, & N_{eDE} \\ N_{eDr}, & N_{err}, & N_{erK}, & N_{erE} \\ N_{eDK}, & N_{erK}, & N_{eKK}, & N_{eKE} \\ N_{eDE}, & N_{erE}, & N_{eKE}, & N_{eEE} \end{bmatrix} \begin{bmatrix} dR_e \\ dr_o \\ dK \\ dE \end{bmatrix} + \begin{bmatrix} W_{eD}+W^*_t \\ W_e \\ W_{eK} \\ W_{eE} \end{bmatrix} = 0 \qquad (3.12)$$

In the above formula (3.12) N^*_{tt} and W^*_t are the reduced normal equations of the previous adjustment of the terrestrial network (see formula (2.22), (2.23) and (2.25). Since all the data now are reduced to the surface of an ellipsoid, the geometrical feature of the data distributions can only determine two shift parameters in equation (3.12). Therefore we have to introduce in appropriate accuracy as compared with that of the other two geodetic coordinates. In case of two dimensional combination, the height errors do not enter into it and will influence only the final transformation parameters for the geocentric parameters, and in the same time its advantage is that it is more easy to be carried out .

dr_o, two surface shift parameters, instead of the three spatial ones (see expression (2.13). One possibility, for example, is to introduce

$$dr_o = G_o \, dt_o \tag{3.13}$$

where

$$G_o = \begin{bmatrix} -\sin B_o \cos L_o \,, & -\sin L_o \\ -\sin B_o \sin L_o \,, & \cos L_o \\ \cos B_o \,, & 0 \end{bmatrix} \tag{3.14}$$

$$dt_o = \begin{bmatrix} M_o \, dB_o \,, & N_o \cos B_o \, dL_o \end{bmatrix}^T \tag{3.15}$$

G_o and dt_o are related to a given point P_o, in most cases, to the initial point of the terrestrial network.

The final coordinates of the common Doppler stations after the combination adjustment are:

$$B^o_{it} + dB_i \,, \; L^o_{it} + dL_i \qquad (i = 1,2,3, \; .....)$$

4. CONCLUSION:

From the two mathematical models of combinations given in paragraph 2 & 3 one should be very careful when deciding which one is better, of course in Egypt till now we don't have enough informations about the geoidal undulations of the terrestrial stations, accordingly if a three dimensional combination is proposed, then one would expect that the errors in the heights will result in the relatively law accuracy in the final coordinates of the combination. This is because the values of the geodetic heights and its weight matrix, which are added into the observations and normal equations (eq. 2.18 and 2.26), do not have an

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